

1. From the list, let's construct the local stellar mass density, using the facts that $L \propto M^{3.5}$ and absolute magnitude $V = V_{\odot} - 2.5 \log_{10} \left(\frac{L}{L_{\odot}} \right)$.

To estimate the local stellar density, let's count up all the mass from these thirty stars, and divide by the volume. The volume first. The farthest star on the list is *Kapteyn's Star*, 12.52 light years away. Recall that one light year is equal to 1 pc/3.26. So the spherical volume we are probing with these thirty stars is

$$\frac{4\pi R^3}{3} = \frac{4\pi(12.52/3.26)^3}{3}(\text{pc})^3.$$

Numerically, I get 237 pc³. If all the stars had the mass of the sun, then the local density would be $30M_{\odot}/237\text{pc}^3 = 0.13M_{\odot}\text{pc}^{-3}$. This is close, but a bit off, because most of the local stars are less massive than the sun. So, let's count the total mass.

Easiest is the sun, which contributes one solar mass. Next is *Proxima Cen*, with absolute magnitude $V = 15.49$. So from above, the ratio of luminosities is

$$\frac{L}{L_{\odot}} = 10^{-0.4(V-V_{\odot})}.$$

In this case, the argument of the exponent on the right hand side is $0.4 * (15.49 - 4.85) = 4.26$ since the absolute visual magnitude of the sun is 4.85. So, $L/L_{\odot} = 5.5 \times 10^{-5}$. Assume that the mass scales as $L^{1/3.5}$; then $M = M_{\odot}(L/L_{\odot})^{2/7}$. In this case, then, $M = M_{\odot}(5.5 \times 10^{-5})^{2/7} = 0.06M_{\odot}$.

The general formula for the mass of a star under these assumptions is then

$$M = M_{\odot}10^{-0.114(V-4.85)}$$

since $0.4 * 2/7 = 0.114$. When I apply this formula to the thirty stars, I get a total mass of 11.84. Add the sun to get 12.84. The local density is then $12.84M_{\odot}/237\text{pc}^3 = 0.05M_{\odot}\text{pc}^{-3}$ in exact agreement with the number given in Carroll and Ostlie on page 932.

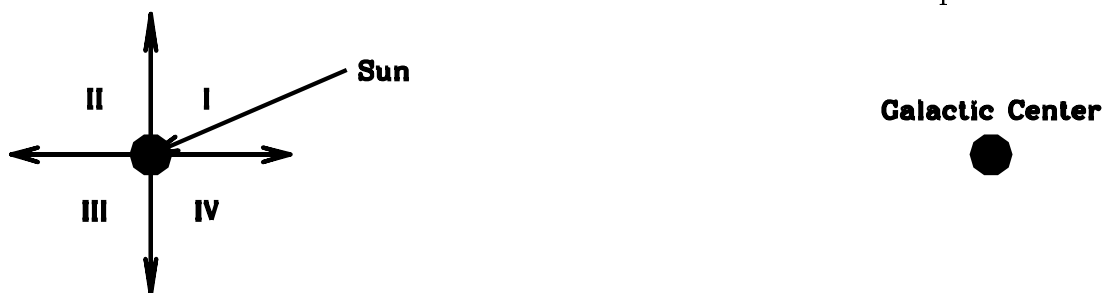
2. (a) Problem 22.2 in CO. Mainly to get you to look at Table 22.1 which is important. For the percentages, I get Thin Disk: $1.8/2.3 = 78\%$; Thick Disk: $.02/2.3 = 0.87\%$; Central Bulge: $0.3/2.3 = 13\%$; and Stellar Halo: $0.1/2.3 = 4.3\%$. A little short of a hundred percent, but if the Stellar halo has a luminosity of 0.14 you can push it up close.

(b) According to footnote c in the Table, the total bolometric luminosity is $3.6 \times 10^{10} L_{\odot}$. Turn this into magnitudes via

$$M = M_{\odot} - 2.5 \log_{10}(L/L_{\odot}).$$

Since $M_{\odot} = 4.76$, I get $M = 4.76 - 2.5 \log(3.6 \times 10^{10}) = -21.63$.

3. Problem 22.16 in CO. Divide the area around the sun into four quadrants



At $l = 0$, pointing towards the Galactic Center, the transverse velocity of the star is its radial velocity minus ours $v - v_0 = -(dv/dR)d = (A + B)d$. In the first quadrant, moving counter-clockwise from $l = 0$ (which points towards the Galactic center) to $l = \pi/2$, $\cos(2l)$ monotonically decreases from 1 to -1 . So the transverse velocity goes from its maximum at $l = 0$ to its minimum at $l = \pi/2$. This makes sense: at $l = \pi/2$, our transverse velocity is zero, while the velocity of the star is not completely perpendicular to the line connecting us, has a small component equal to $\sin \alpha = -(d/R)$ times its circular velocity. (See figure 22.24 in the book.) In the second quadrant, our velocity dominates since we're closer to the center. But the transverse component of our velocity in this quadrant is positive (towards the upper right). Since we *subtract* our velocity from the other guy's, the net transverse velocity is negative. Same holds true in quadrant III: the transverse component of our velocity is pointing to the upper left (i.e. is positive); when we subtract it off, we get a negative answer. In quadrant IV, the star's velocity is most important, and it is negative (points up), so again the net transverse velocity is negative.

4. Problem 24.1 CO. (a) Take the mass of an M star to be $0.5M_{\odot}$. Then, the number density is the mass density divided by this:

$$n = \frac{0.05M_{\odot}\text{pc}^{-3}}{0.5M_{\odot}} = 0.1\text{pc}^{-3}$$

The total number of stars in the Galactic disk is therefore nV_{disk} . The total volume occupied by these stars is this number times the volume of a single star. Take the radius of an M star to be $0.6R_{\odot} = 0.6 \times 6.96 \times 10^{10} \text{ cm} = 4.2 \times 10^{10} \text{ cm}$. The total volume of the stars is therefore

$$V_{\text{stars}} = 0.1 \text{ pc}^{-3} V_{\text{disk}} \frac{4\pi(4.2 \times 10^{10} \text{ cm})^3}{3}$$

The fraction of the disk volume occupied by stars is simply this divided by V_{disk} , so

$$f_{\text{star}} = 0.1 \text{ pc}^{-3} \frac{4\pi(4.2 \times 10^{10} \text{ cm})^3}{3}$$

But one parsec is equal to $3 \times 10^{18} \text{ cm}$, so

$$f_{\text{star}} = 1.1 \times 10^{-24},$$

small.

(b) The chance of hitting a star is equal to

$$\frac{(\text{Area occupied by all stars})}{(\text{Total Area of Disk})}$$

where by *area*, I mean you project all stars at a given R, ϕ onto a 2D disk no matter where they are in z . First, let's do the denominator. It is just πR_{disk}^2 , the area of a circle. It will be useful to rewrite this as the volume of the disk divided by its height, $V_{\text{disk}}/h_{\text{disk}}$. The numerator is the area occupied by a single star times the total number of stars. The area occupied by a spherical star though is πR_{star}^2 , or its volume divided by $4R_{\text{star}}/3$. Therefore,

$$\text{Area occupied by all stars} = \frac{V_{\text{star}} \text{ Number of Stars}}{(4R_{\text{star}}/3)}.$$

So the chances of hitting a star are

$$\frac{V_{\text{star}} \text{ Number of Stars}}{(4R_{\text{star}}/3)} \frac{h_{\text{disk}}}{V_{\text{disk}}} = \left(\frac{V_{\text{star}} \text{ Number of Stars}}{V_{\text{disk}}} \right) \frac{3h_{\text{disk}}}{4R_{\text{star}}}.$$

But the term in parentheses is precisely equal to the fraction f we computed in part (a).

So, plugging in numbers, I get odds of

$$1.1 \times 10^{-24} \frac{3 \text{ kpc}}{4 \times 4.2 \times 10^{10} \text{ cm}} = 6.1 \times 10^{-14}.$$